Introduction to Trace

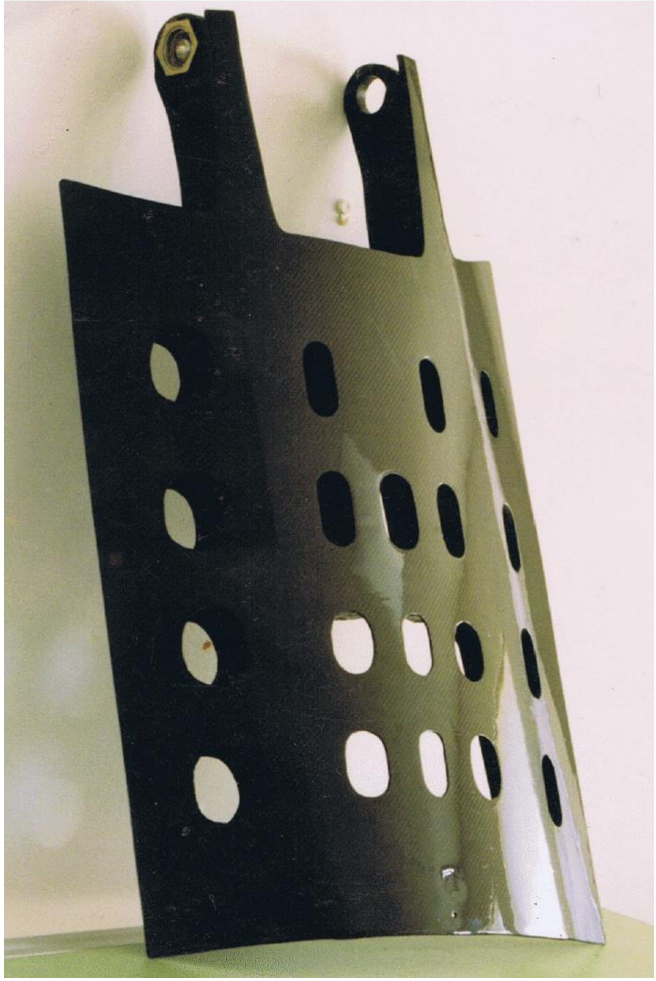
Dan Retief PhD

Aero Design Limited

Composites Design Challenge

Simplify the way we deal with material properties and make it as easy as working with Aluminium.

Lets see where we were before:



Black Aluminium

Design it as if it is aluminium or replace existing aluminium component:

Extensive Use of Quasi Isentropic

Used rules of thumb to design equivalent laminates

Use large factors of safety

Test Outcomes

Strength and stiffness determined by test.

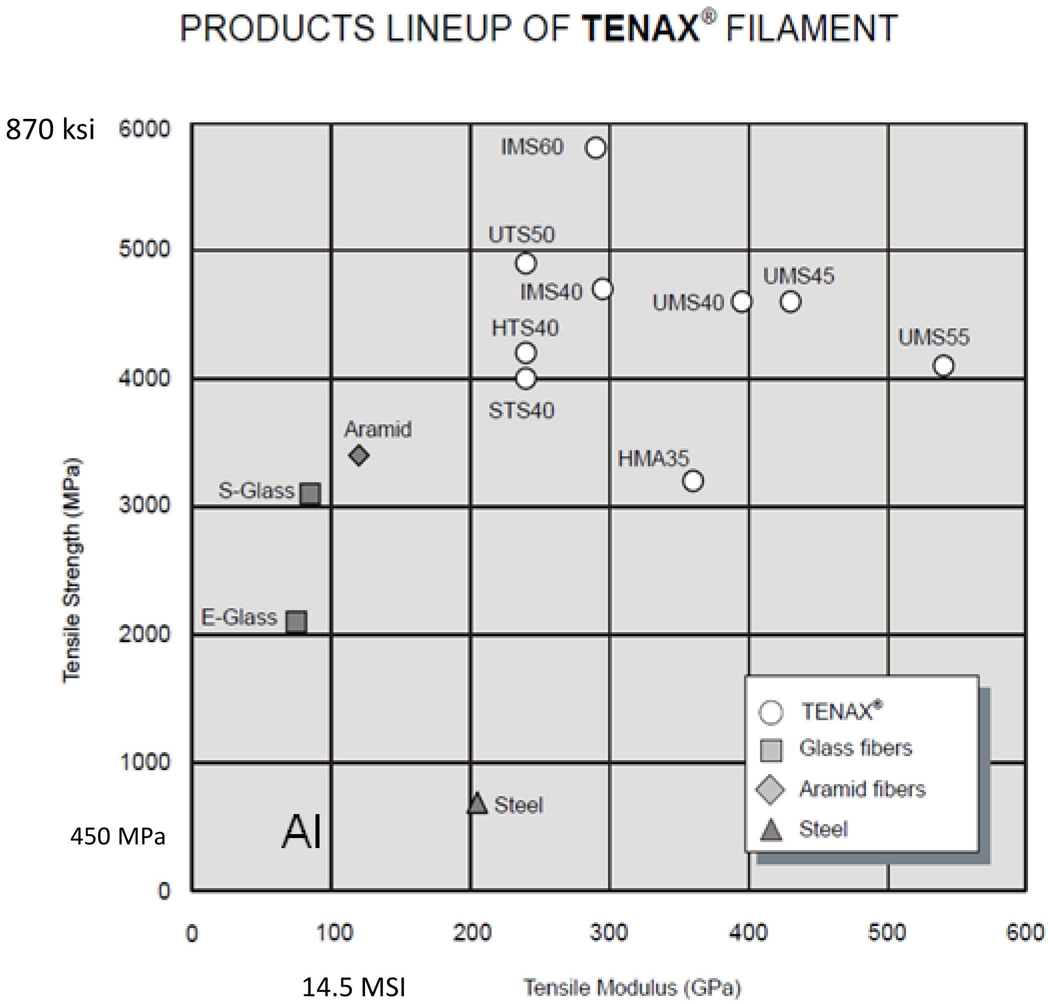
Weight similar to aluminium.

Sometimes too heavy.

Stiffness barely better than Aluminium.

Cost too high to redesign and retest.

What is the point?



The Potential

Carbon Filaments with stiffness of steel

Strength over 10X that of aluminium

Stiffness over 2X that of aluminium

Corrosion free

Fatigue Resistant

Efficiency of Sandwich Structures

Some challenges:

Temperature

Moisture

Damage Tolerance & resistance

Prototype Cargo Pod

Ultimate Static Test > 6000 lb

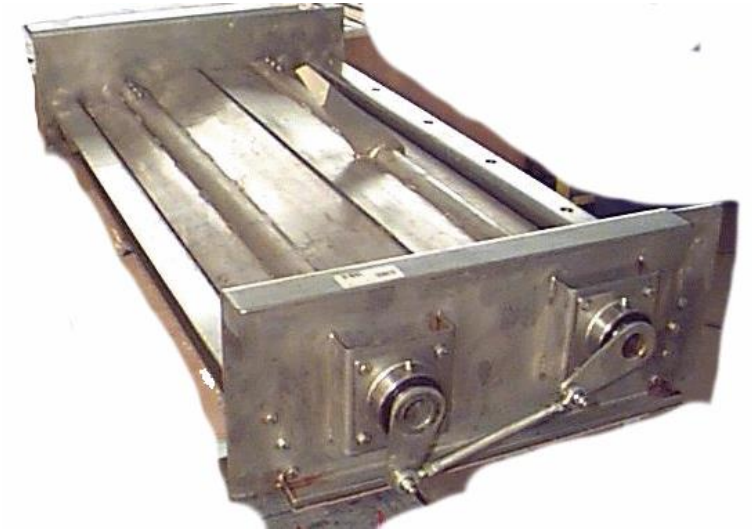
Some success

- Sandwich Construction
- Weight of pod 45kg
- Loaded with over 6000 lb of lead ingots
- No failure, high stiffness
- High strength to weight ratio
- Achieved with glass fibre facings on Nomex Honeycomb core

The potential is there…

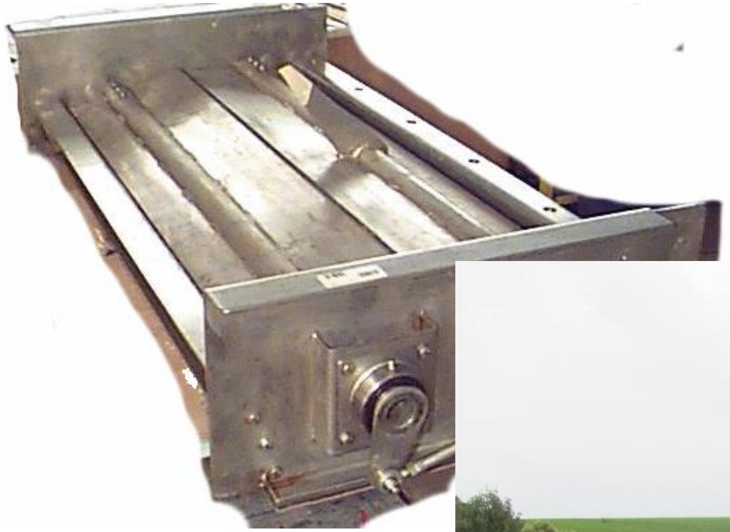
Hopper Gate Box

Steel – 65kg



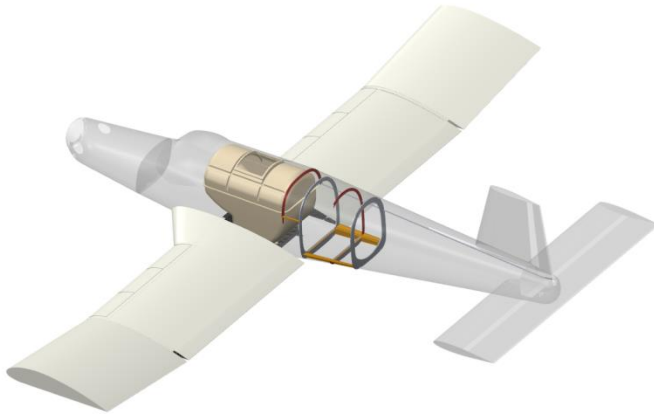
Carbon – 15kg

Steel – 65kg

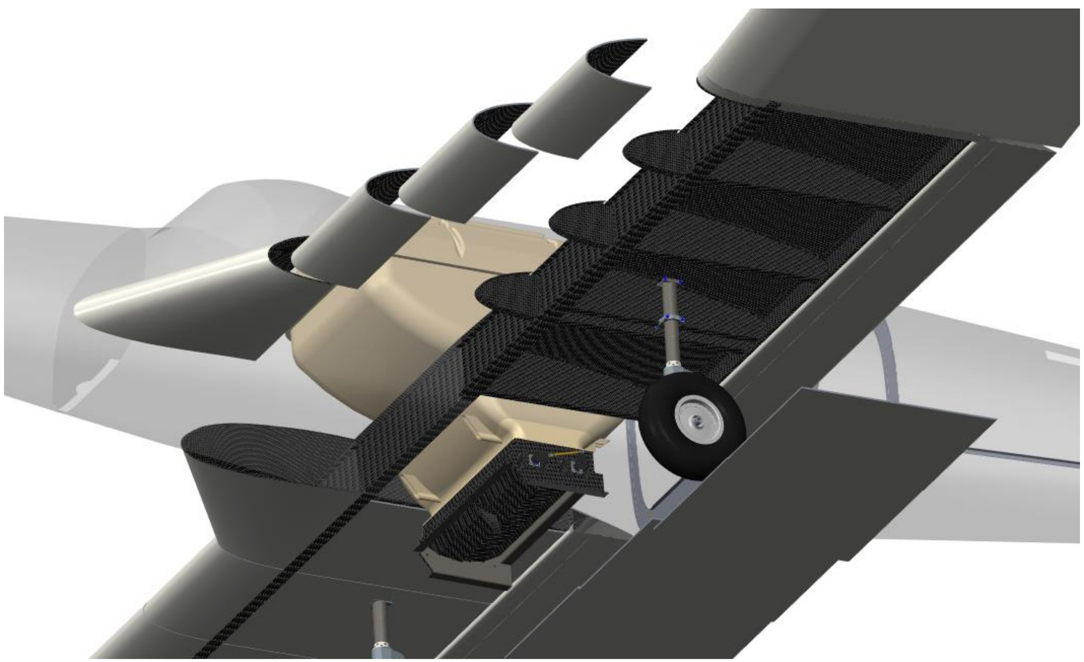


Hopper Gate Box

Carbon – 15k



Cresco Carbon Composite Centre Wing Section



New Project,

New Design Challenges

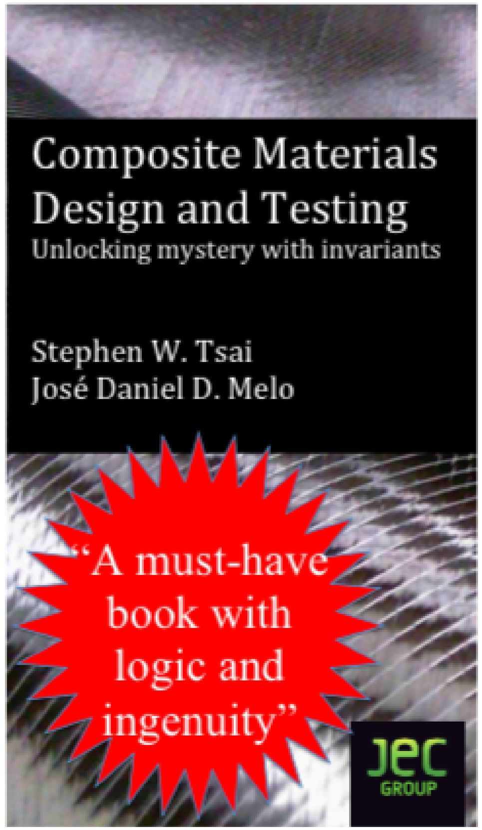
Trace to the rescue!

Beyond Black Aluminium

Online, Live Composites Design Workshop X

Stephen W. Tsai & Associates

July 13-17, 2015; noon to 4 PM PST – USA



- **Truly authoritative and practical.**
- **20 hours of live online sessions.**
- **Presents a totally new invariant-based approach to stiffness and strength and the concept of homogenization for optimal design. Much simplified.**
- **Sessions including tools and case studies are downloadable for later individual viewing.**
- **US$1,500 registration* covers: the new book *Composite Materials Design and Testing*, two e-books, tools, composites app iMicMac and iPad Air 2.**
- **Optional official transcript of records and certificate from Stanford University**.**

For information on ordering book and workshop registration:
http://compositesdesign.stanford.edu

* The registration fee is $1000* for participants living outside the USA and does not include iPad.
** Continuing Studies courses carry Continuing Education Units (CEU's), not undergraduate/graduate credits. Credits cannot be applied toward any Stanford degree. Credits will be recorded on your transcript and you might be able to transfer the credits to another university, subject to that university's policies.

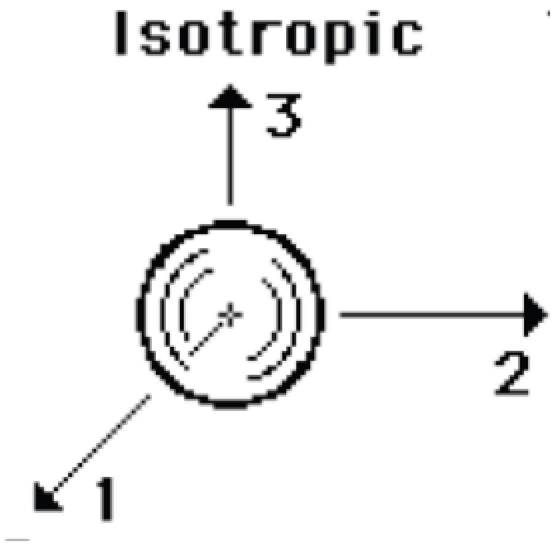
LAMINATED PLATE THEORY: Stiffness

Composites Design Workshop X

Stephen W. Tsai

Stanford University

July 13, 2015



Hooke’s Law

In matrix form, Hooke's law for isotropic materials can be written as:

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\\ 2\varepsilon_{12}\end{bmatrix} = \begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0\\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu)\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix}$$

where $\gamma_{ij} := 2\varepsilon_{ij}$ is the **engineering shear strain**. The inverse relation may be written as

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ \nu & 1-\nu & \nu & 0 & 0 & 0\\ \nu & \nu & 1-\nu & 0 & 0 & 0\\ 0 & 0 & 0 & (1-2\nu)/2 & 0 & 0\\ 0 & 0 & 0 & 0 & (1-2\nu)/2 & 0\\ 0 & 0 & 0 & 0 & 0 & (1-2\nu)/2\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\\ 2\varepsilon_{12}\end{bmatrix}$$

Only two independent variables: E, ν

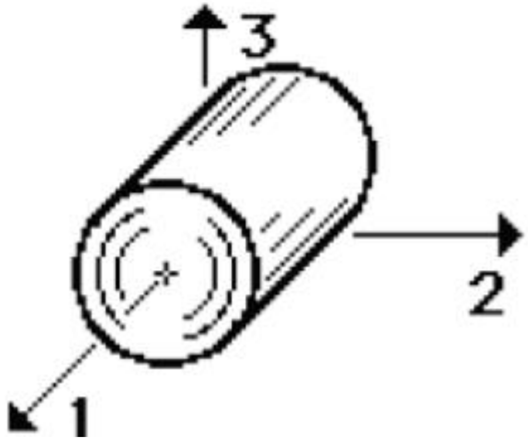
Under plane stress $\sigma_{31} = \sigma_{13} = \sigma_{32} = \sigma_{23} = \sigma_{33} = 0$

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}$$

Or in inverse form the familiar:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}$$

Transverly Isotropic



The material matrix $\underline{\underline{\boldsymbol{K}}}$ has a symmetry with respect to a given orthogonal transformation ($\boldsymbol{A}$) if it does not change when subjected to that transformation. For invariance of the material properties under such a transformation we require

$$\boldsymbol{A}\cdot\mathbf{f} = \boldsymbol{K}\cdot(\boldsymbol{A}\cdot\boldsymbol{d}) \implies \mathbf{f} = (\boldsymbol{A}^{-1}\cdot\boldsymbol{K}\cdot\boldsymbol{A})\cdot\boldsymbol{d}$$

Hence the condition for material symmetry is (using the definition of an orthogonal transformation)

$$\boldsymbol{K} = \boldsymbol{A}^{-1}\cdot\boldsymbol{K}\cdot\boldsymbol{A} = \boldsymbol{A}^{T}\cdot\boldsymbol{K}\cdot\boldsymbol{A}$$

Orthogonal transformations can be represented in Cartesian coordinates by a 3×3 matrix $\underline{\underline{\boldsymbol{A}}}$ given by

$$\underline{\underline{\boldsymbol{A}}} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} .$$

Therefore the symmetry condition can be written in matrix form as

$$\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{A}^{T}}}\ \underline{\underline{\boldsymbol{K}}}\ \underline{\underline{\boldsymbol{A}}}$$

For a transversely isotropic material, the matrix $\underline{\underline{\boldsymbol{A}}}$ has the form

$$\underline{\underline{\boldsymbol{A}}} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

In linear elasticity, the stress and strain are related by Hooke's law:

$$\underline{\underline{\boldsymbol{\sigma}}} = \underline{\underline{\mathsf{C}}}\ \underline{\underline{\boldsymbol{\varepsilon}}}$$

or, using Voigt notation

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

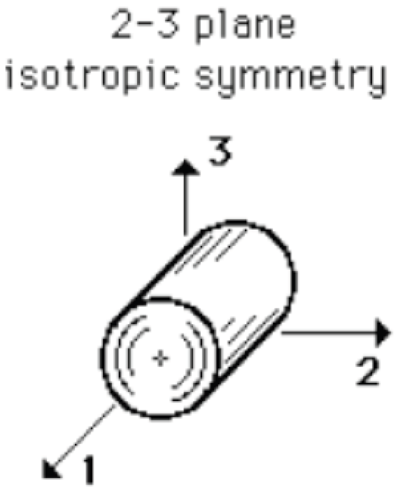
The condition for material symmetry in linear elastic materials is

$$\underline{\underline{\mathsf{C}}} = \underline{\underline{\mathsf{A}_\varepsilon}}^T\ \underline{\underline{\mathsf{C}}}\ \underline{\underline{\mathsf{A}_\varepsilon}}$$ giving:

$$\underline{\underline{\mathsf{C}}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & (C_{11}-C_{12})/2 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{11}-2C_{66} & C_{13} & 0 & 0 & 0 \\ C_{11}-2C_{66} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}.$$

Stiffness and Compliance Matrices

Figure 1.5



Isotropic-plane (2 constants)

$$
\begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{21} & C_{32} & C_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{C_{22}-C_{23}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\qquad
\begin{bmatrix}
S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\
S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{21} & S_{32} & S_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{S_{22}-S_{23}}{1/2} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
$$

Matrix inversion →

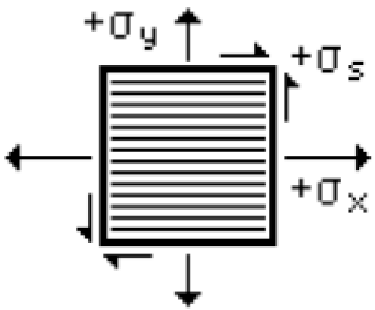
Engineering constants

In engineering notation,

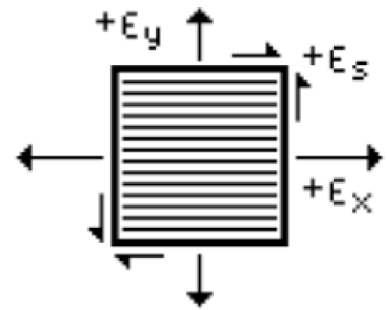
$$\underline{\underline{C}}^{-1} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_x} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_x} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{xz}}{E_x} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu_{xy})}{E_x} \end{bmatrix}$$

Note we have 5 variables, compared with 2 in isotropic material.

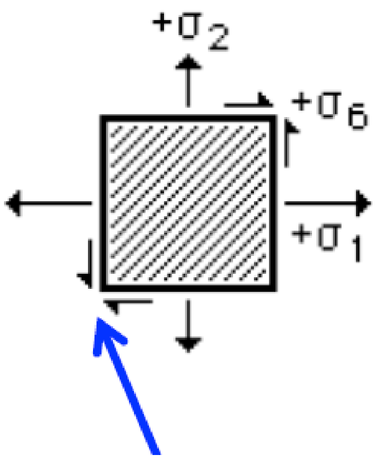
In the plane stress case for stiffness |Q|



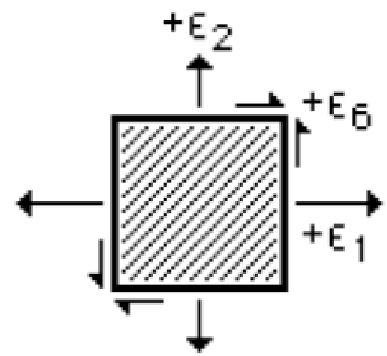
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{yx} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_s \end{Bmatrix}$$



On-axis stress-strain relation

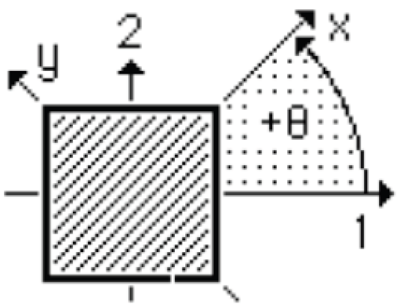


$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix}$$



Off-axis stress-strain relation

Plane Stress Stiffness Transformation



$$
\begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{Bmatrix}
=
\left[\begin{array}{cccc}
m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\
n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\
m^2n^2 & m^2n^2 & m^4+n^4 & -4m^2n^2 \\
m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2)^2 \\
\hline
m^3n & -mn^3 & mn^3-m^3n & 2(mn^3-m^3n) \\
mn^3 & -m^3n & m^3n-mn^3 & 2(m^3n-mn^3)
\end{array}\right]
\begin{Bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{xy} \\ Q_{ss} \end{Bmatrix}
$$

Figure 2.4. TRANSFORMATION OF STIFFNESS MATRIX FOR POSITIVE ANGLES
($m = \cos\theta$, $n = \sin\theta$)

Note: even power above blue line; odd power below it

Linear Combinations of [Q]

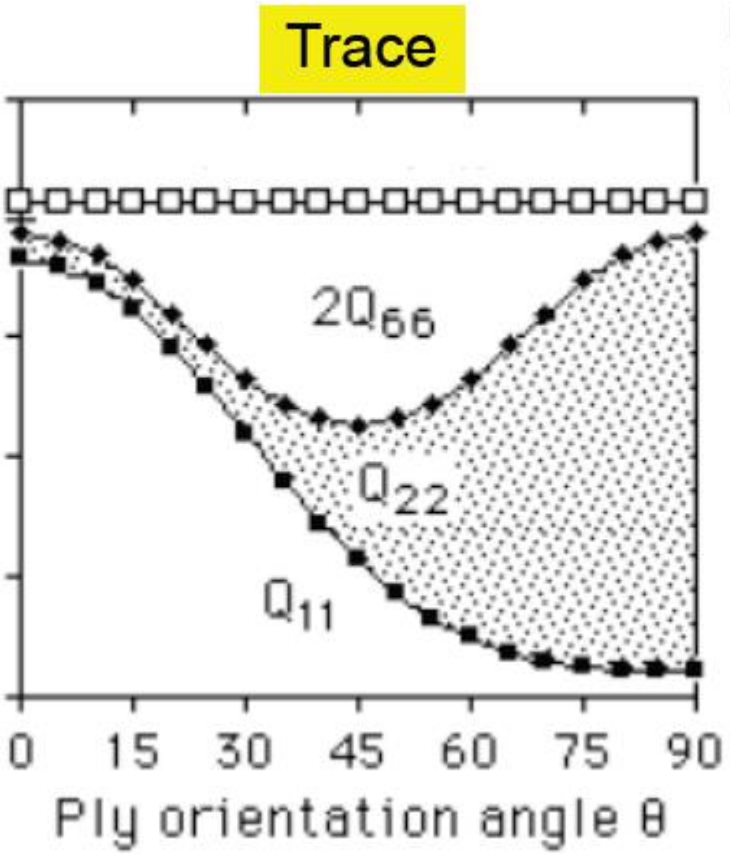
$$m^4 = \frac{3+\cos 2\theta+\cos 4\theta}{8}, \quad m^3 n = \frac{2\sin 2\theta+\sin 4\theta}{8}$$

$$m^2 n^2 = \frac{1-\cos 4\theta}{8}, \quad mn^3 = \frac{2\sin 2\theta-\sin 4\theta}{8}, \quad n^4 = \frac{3-4\cos 2\theta+\cos 4\theta}{8} \qquad (3.7)$$

Table 2.1 LINEAR COMBINATIONS OF ON-AXIS STIFFNESS MODULI

	Q_{xx}	Q_{yy}	Q_{xy}	Q_{ss}	Invariant?
$U_1 = U_4+2U_5$	3/8	3/8	1/4	1/2	Yes
U_2	1/2	-1/2	0	0	No
U_3	1/8	1/8	-1/4	-1/2	No
$U_4 = U_1-2U_5$	1/8	1/8	3/4	-1/2	Yes
$U_5 = (U_1-U_4)/2$	1/8	1/8	-1/4	1/2	Yes

Two independent invariants: $U_1 = U_4 + 2U_5$



Trace = $Q_{11}+Q_{22}+2Q_{66}$

Trace does not vary with ply orientation.

Master Ply

Material	E_x (GPa)	E_y (GPa)	ν_x	E_s (GPa)	Q_{xx}*	Q_{yy}*	Q_{xy}*	Q_{ss}*	Tr (GPa)	E_x*
IM6/epoxy	203	11.20	0.32	8.40	0.8791	0.0485	0.0155	0.0362	232	0.874
IM7/977-3	191	9.94	0.35	7.79	0.8825	0.0459	0.0161	0.0358	218	0.877
T300/5208	181	10.30	0.28	7.17	0.8805	0.0501	0.0140	0.0347	206	0.877
IM7/MTM45	175	8.20	0.33	5.50	0.9014	0.0422	0.0139	0.0282	195	0.897
T800/Cytec	162	9.00	0.40	5.00	0.8955	0.0497	0.0199	0.0274	183	0.888
IM7/8552	159	8.96	0.32	5.50	0.8888	0.0501	0.0160	0.0306	180	0.884
T800S/3900	151	8.20	0.33	4.00	0.9034	0.0491	0.0162	0.0238	168	0.898
T300/F934	148	9.65	0.30	4.55	0.8878	0.0579	0.0174	0.0271	168	0.883
T700 C-Ply 64	141	9.30	0.30	5.80	0.8713	0.0575	0.0172	0.0356	163	0.866
AS4/H3501	138	8.96	0.30	7.10	0.8567	0.0556	0.0167	0.0438	162	0.852
T650/epoxy	139	9.40	0.32	5.50	0.8724	0.0590	0.0189	0.0343	160	0.866
T4708/MR60H	142	7.72	0.34	3.80	0.9029	0.0491	0.0167	0.0240	158	0.897
T700/2510	126	8.40	0.31	4.20	0.8827	0.0588	0.0182	0.0292	144	0.877
AS4/MTM45	127	7.93	0.30	3.60	0.8938	0.0558	0.0167	0.0252	143	0.889
T700 C-Ply 55	121	8.00	0.30	4.70	0.8746	0.0578	0.0173	0.0338	139	0.869
Std dev	24.6	1.0	0.029	1.5	0.0132	0.0053	0.0016	0.0056		0.013
Coeff var %	16.0	10.9	9.0	27.2	1.5	10.1	9.6	17.9		1.5
Master ply					**0.8849**	**0.0525**	**0.0167**	**0.0313**		**0.880**

1) Invariant stiffness

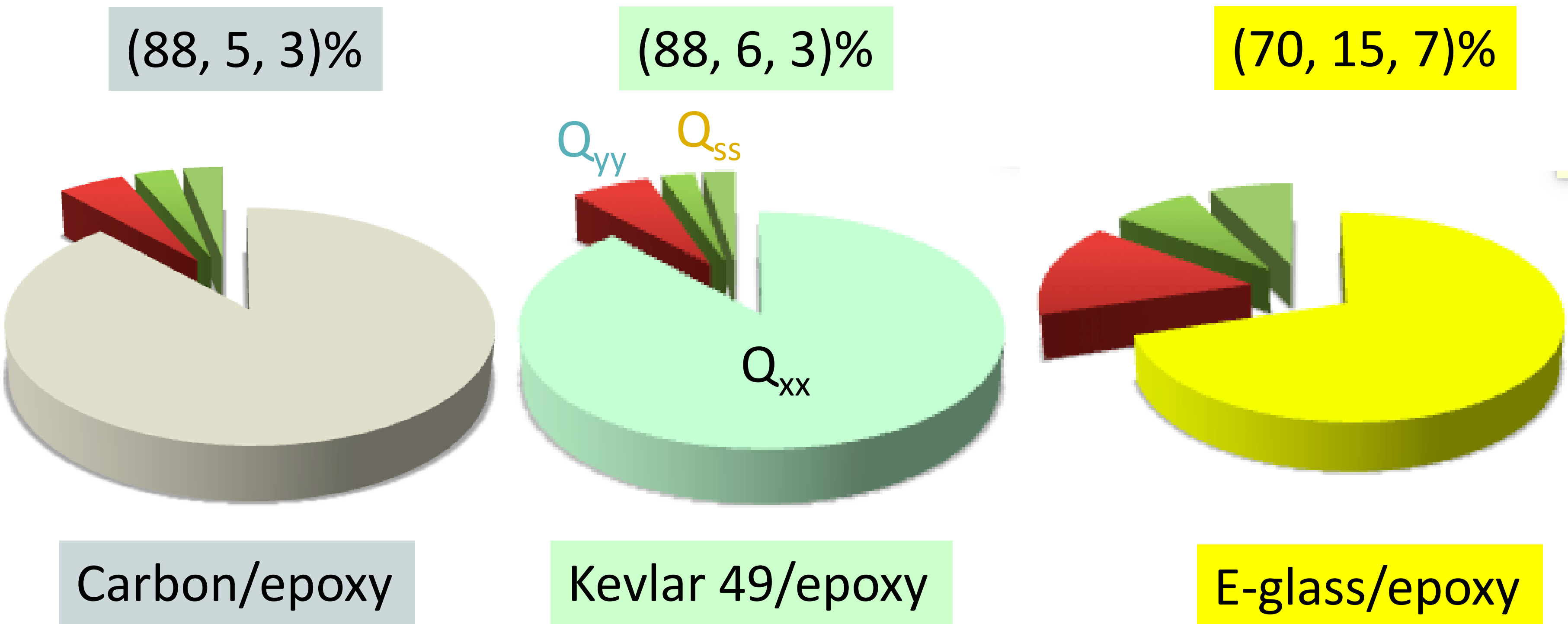
$$Tr\,[Q] = Q_{11} + Q_{22} + 2Q_{66}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 2Q_{16} \\ Q_{21} & Q_{22} & 2Q_{26} \\ Q_{61} & Q_{62} & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix}$$

Need only ONE stiffness E_x

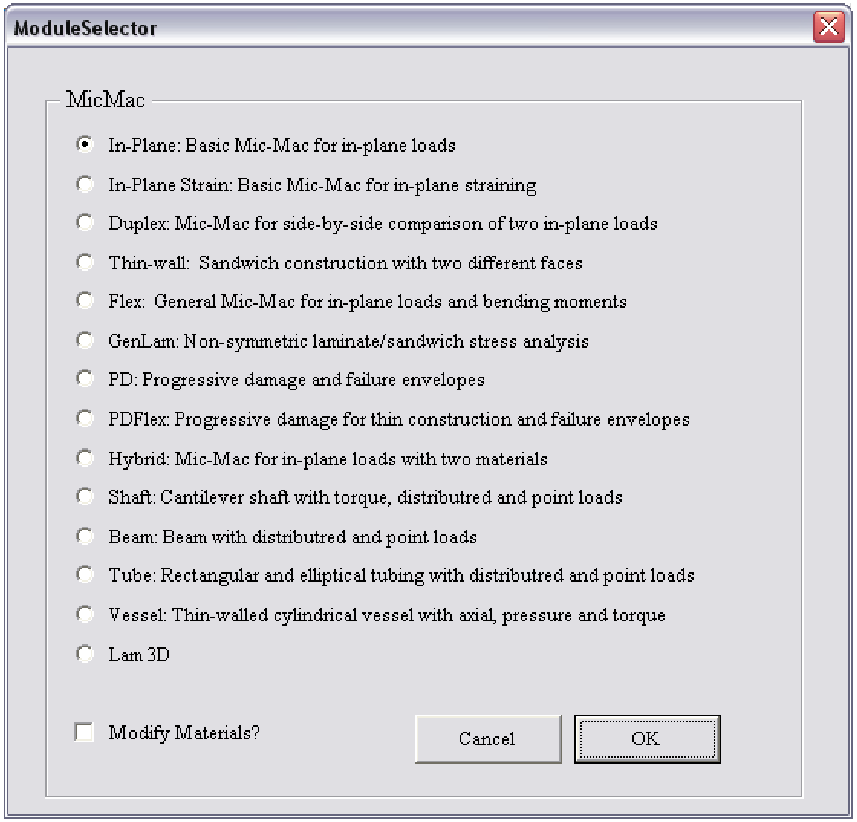
$$Tr\,[Q] = \frac{E_x}{0.88}$$

Composition of Trace: Carbon, Kev, Glass



Ply material	Q_{xx}*	Q_{yy}*	Q_{ss}*	Q_{xy}*	Trace
Average CFRP	0.88	0.05	0.03	0.02	1.000
Kevlar/epoxy	0.88	0.06	0.03	0.03	1.000
E-glass/epoxy	0.70	0.15	0.07	0.04	1.000

MicMac Spreadsheet tool



Calculate macroscopic material properties for any laminate and orientation from microscopic material properties.

How do we get the micro properties

Testing – As Manufactured

Material Manufacturer’s Data

Mechanical Testing of Composites

Composites Design Workshop X

July 16, 2015

Jared W. Nelson

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Alan T. Nettles

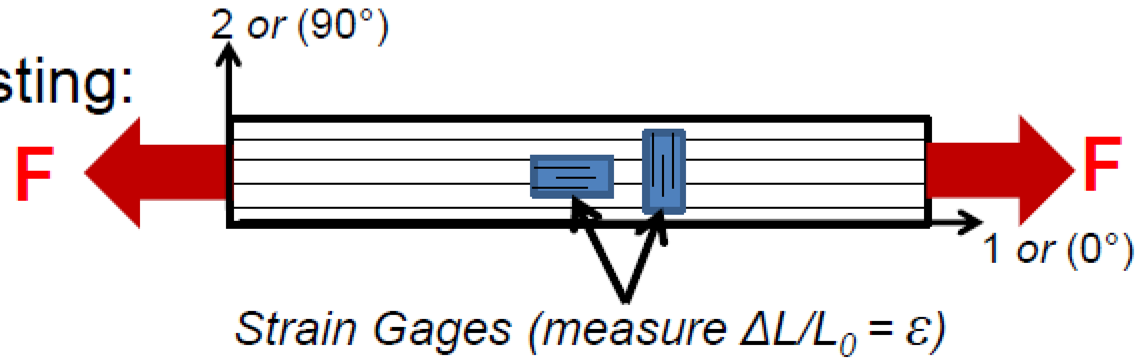
Stanford University, Stanford, CA

NASA Marshall Space Flight Center, MSFC, AL

Tensile Test

Lamina (fibers in one direction) Tensile Testing:

Longitudinal Testing:



A=cross-sectional area of specimen = width X thickness

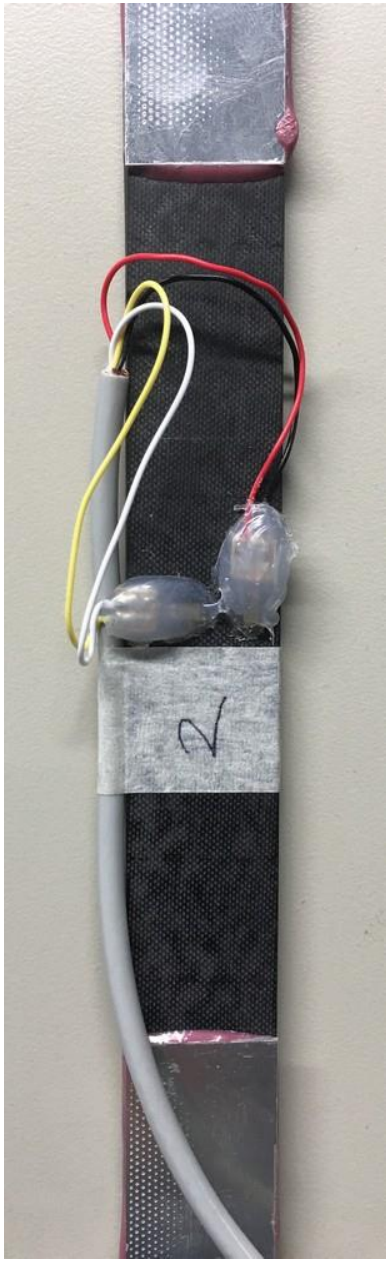
Properties typically generated:

0° Tensile Strength $\sigma_1^{tu} = F^u/A$

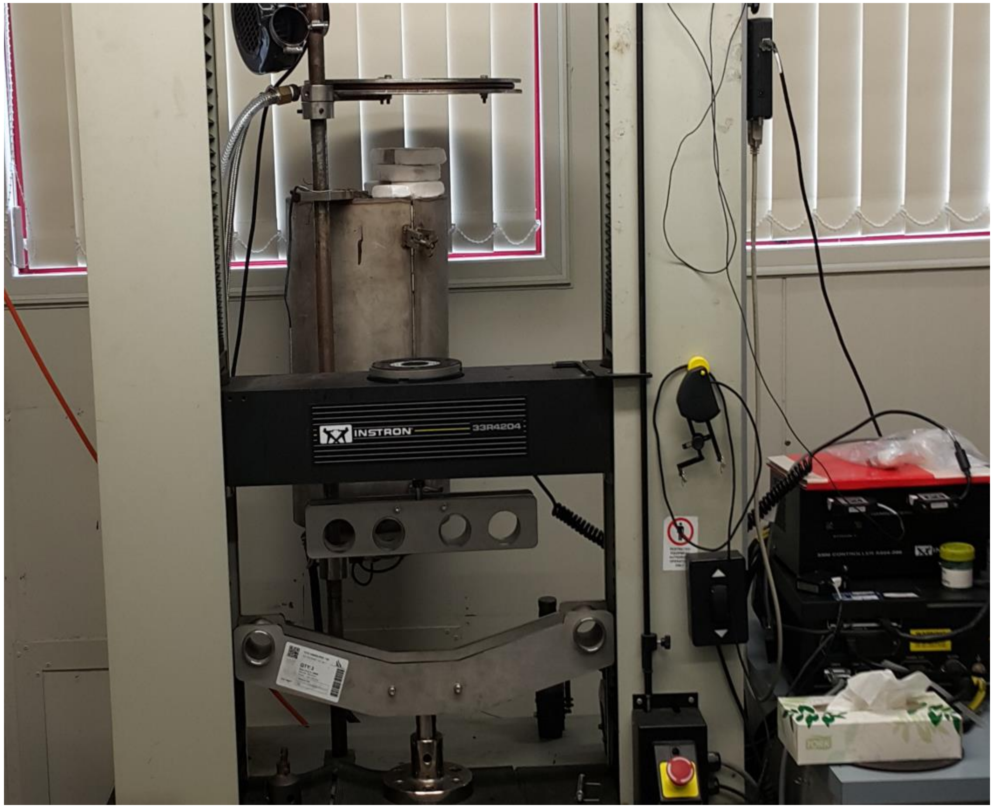
Difficult to measure (More on this later)

Longitudinal Tensile Modulus $E_1^t = \sigma_1/\varepsilon_1$

Poisson's Ratio $\nu_{12} = -\varepsilon_2/\varepsilon_1$ *~ 0.3*



Unidirectional coupon with 0/90 strain gauges and aluminium grips

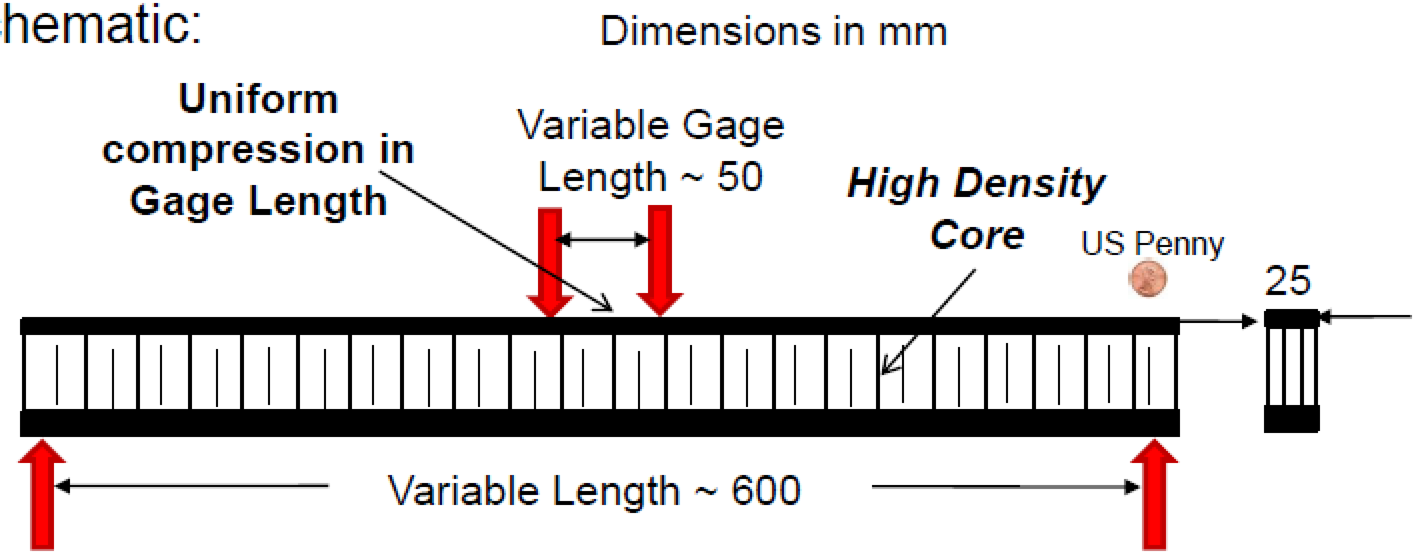


Four Point Beam Test

Compression Testing Specifics:

ASTM D-5467: Sandwich Beam Method

Schematic:



Pros:

- Eliminates buckling of laminate
- Strain Gages not needed
- Needs much lower loads to break

Cons:

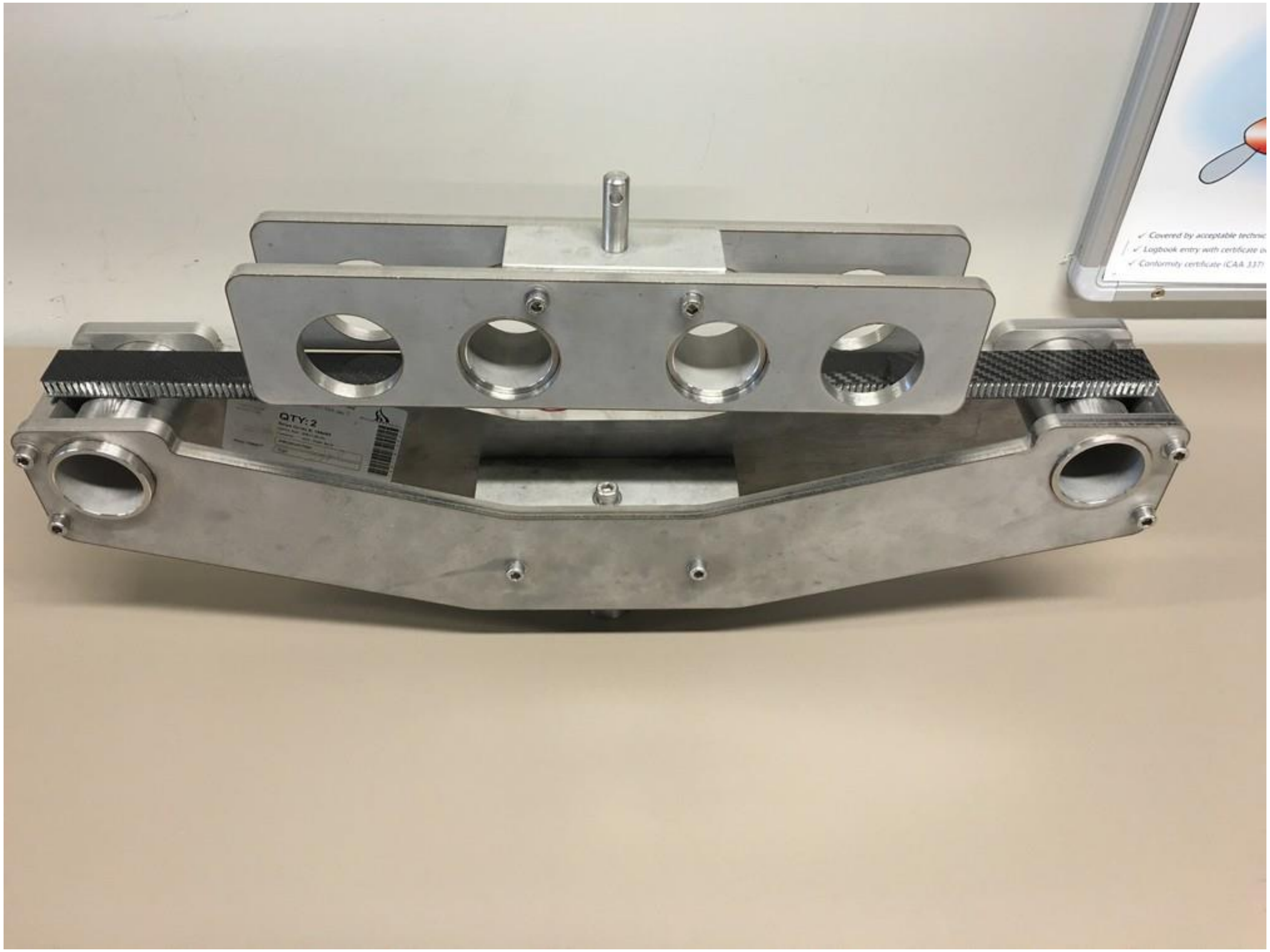
- Slight Variation in strain through the laminate thickness (~10%)
- Specimens are quite expensive
- Bottom Face Sheet needs to be sized so tensile failures do not occur

Test Beam

0/90 Carbon Facings

Aluminium Honeycomb Core

Strain gauges 0/90


QTY: 2

Design Allowables

How do we calculate design allowables from test data

Structural Reliability Theory

and

Software Tools

for

Composites Design (*)

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Applied & Computational Mathematics Division
NIST, Gaithersburg, MD 20899-8910
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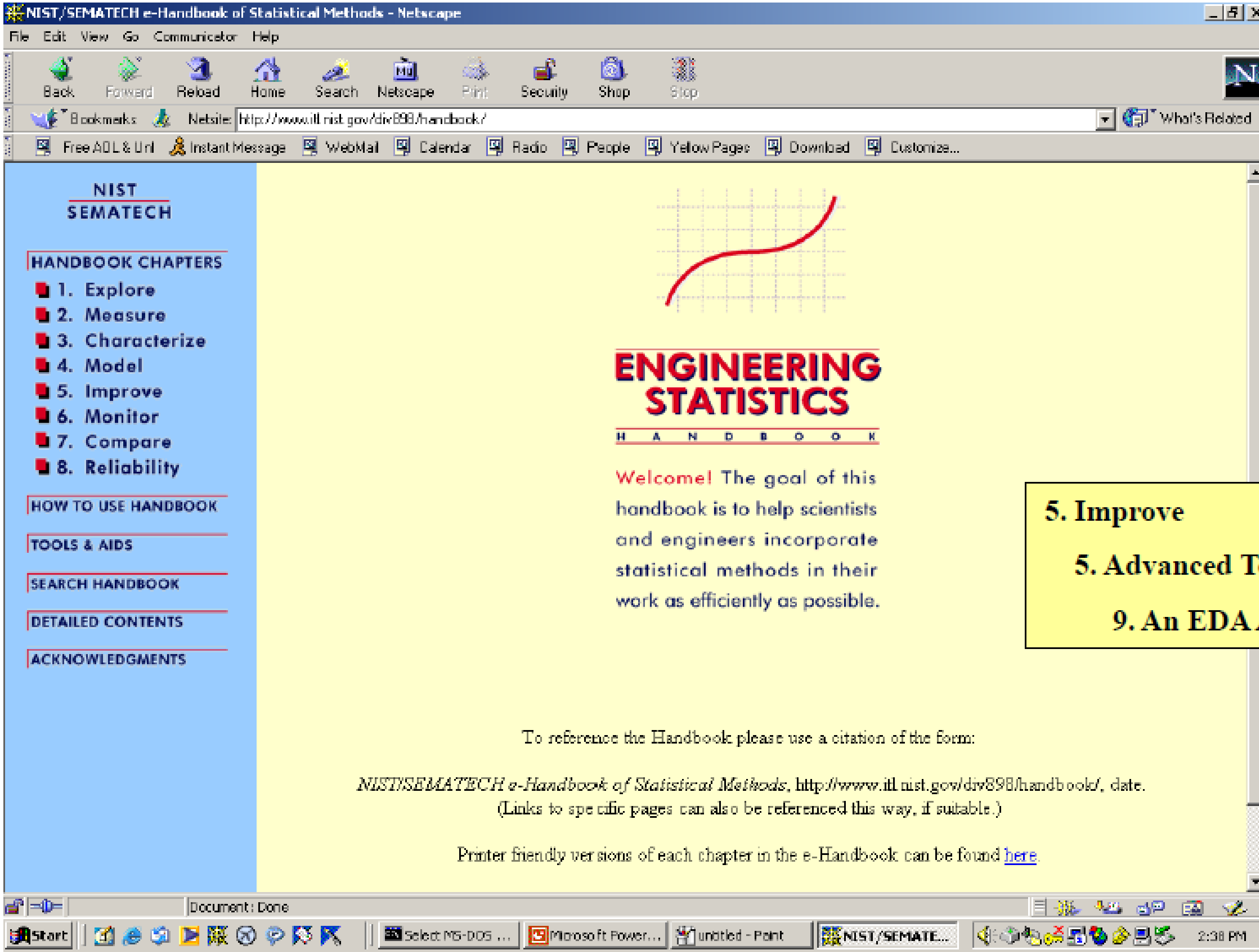
NIST e-Handbook of Engineering Statistics



5. Improve

5. Advanced Topics

9. An EDA Approach to DE

http://www.itl.nist.gov/div898/handbook/ (3000 pages; 3 million page views / month)

Mechanical Testing of Composites

Composites Design Workshop X

July 16, 2015

Jared W. Nelson

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SUNY New Paltz, New Paltz, NY

Alan T. Nettles

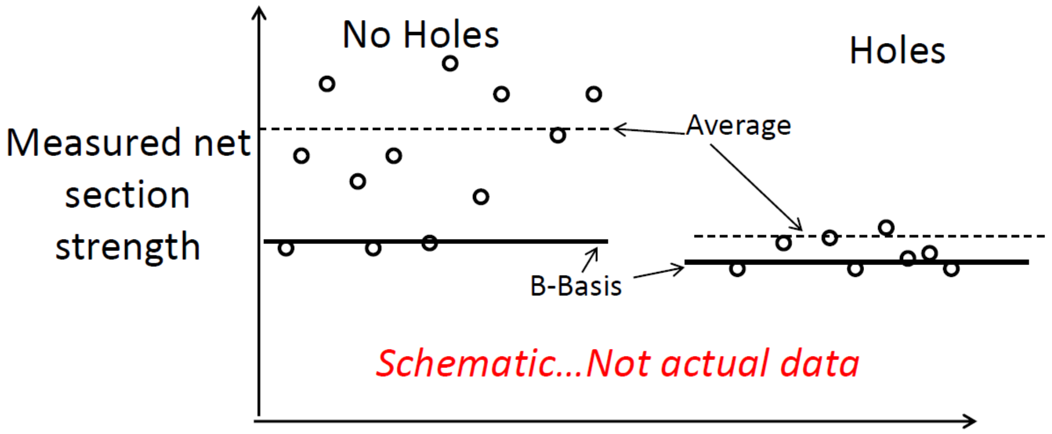
Stanford University, Stanford, CA

NASA Marshall Space Flight Center, MSFC, AL

A and B basis design allowables depends on test data statistics (JW Nelson)

Comment on Testing

Specimens with holes can have nearly the same B-Basis allowable as specimens with no hole!*



* First noted to author by by A. Hodge

Our experience with testing

- As manufactured test samples are a good measure of manufacturing process capability – measure against material manufacturer's data.
- Compression strength of carbon laminates depends highly on configuration – failure mode is buckling of the fibre and depends on lateral support for the fibre.
- Nomex honeycomb core can only support relatively thin carbon fibre face sheets effectively, Al honeycomb is better and can be used in testing.

Online, Live Composites Design Workshop XII

June 20-25, 2016; noon to 4 PM PDT; 20 hours + homework

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For info/registration: http://compositesdesign.stanford.edu